HELM

An outline

Problem Statement

- The equations
  - (1)

$$\sum_{a\in\{all\}} Y_{na}V_a = y_n^{(zip)}V_n + I_n^{(zip)} + \frac{S_n^*}{V_n^*}$$

$$n \in \{all \setminus swing\}$$

$$\frac{S_{sw}^*}{V_{sw}^*} = -y_{sw}^{(zip)}V_{sw} - I_{sw}^{(zip)} + \sum_{a\in\{all\}} Y_{sw,a}V_a$$

- The unknowns
  - (2)

$$V_n = \mathrm{Re}(V_n) + j\,\mathrm{Im}(V_n) = |V_n|e^{j\theta};$$

$$n \in \{all \setminus swing\}$$

$$V_{sw} = |V_{sw}|e^{j\theta_{sw}}$$

Space of Solutions

- Multi-valued
  - Depending on the problem data:
    - Values of the coefficients of the impedance matrix Y_{ij}
    - Injections $I_n, S_n^*, ..$

  The problem has in general a number of solutions of the order of 2^N (for $N+1$ buses).

- What matters
  - Is there a solution to the posed problem or not?.
  - If the problem has solutions, which solution is the one that materializes on the actual physical network?.

  (for a toy grid with 20 nodes, the number of solutions amounts to 1,048,576).

Holomorphic Embedding

- Defining the embedding
  - The unknowns V_n become functions of a complex parameter s.
  - Note that a factor s is introduced for the non-linear power factor contributions.
  - Note that a new function is introduced: $\overline{V}_n(s) \equiv V_n^*(s^*)$. This allows the embedding to be holomorphic. (For real s, one must have $\overline{V}_n(s) = V_n^*(s)$)
- Important Remark
  - At s=0, this system has no injections
  - Provided that the admittance matrix is non singular, the real world solution in this case is:
    - no flows
    - all voltages equal to V_{sw}

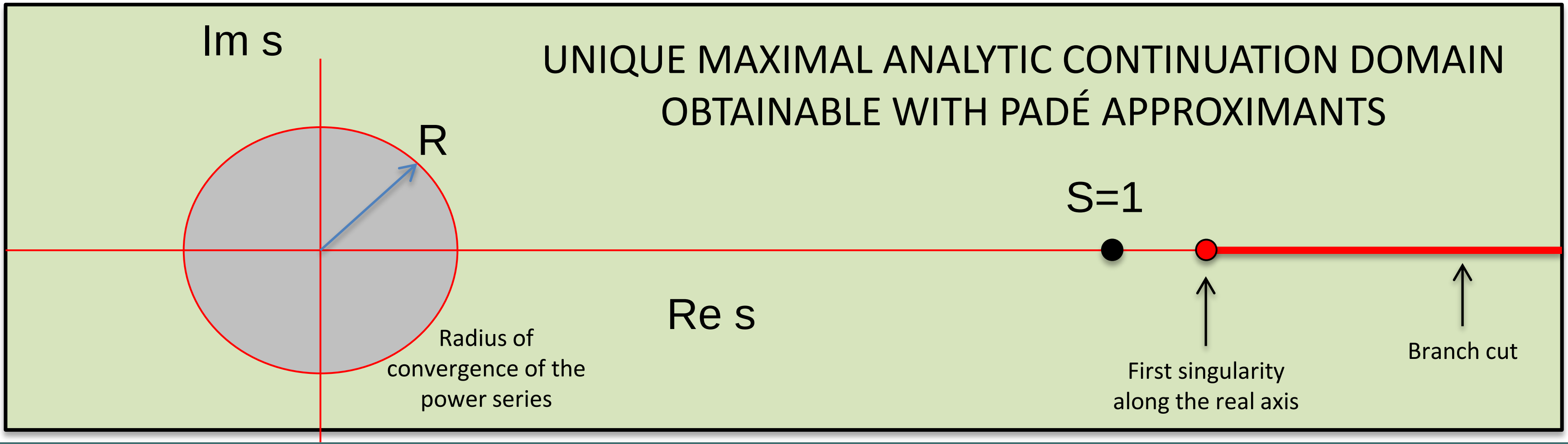
$$\sum_{\alpha \in \{all\}} Y_{n\alpha} V_\alpha(s) = y_n^{(zip)} V_n(s) + I_n^{(zip)} + \frac{s S_n^*}{\overline{V}_n(s)}$$

$$n \in \{all \setminus swing\}$$

$$\frac{S_{sw}^*}{V_{sw}^*} = - y_{sw}^{(zip)} V_{sw} - I_{sw}^{(zip)} + \sum_{\alpha \in \{all\}} Y_{sw,\alpha} V_\alpha(s)$$

Rigorous Results

- The holomorphic embedding + the algebraic structure of the equations guarantee:
  - The problem has always a solution in a finite neighborhood of s=0 (provided that $Y_{nn'}$ is non singular).
- What can be said outside this domain of convergence:
  - Stahl's Theorem (1997), guarantees that holomorphic functions with branch points (the solutions of the embedded power flow problem are such kind of functions) have a maximal domain of analytical continuation, given by Padé Approximants.



Construction of the Solution

Embedded power flow equations + operational solution at s=0.

$$\sum_{n'\in\{nodes\}} Y_{nn'}V_{n'}(s) = -Y_{n,sw}\underbrace{V_{sw}}_{const} + s y_n^{(zip)} V_n(s) + \underbrace{I_n}_{const} + s\frac{S_n^*}{\bar{V}_n(s)} \quad : \quad n\in\{nodes\}$$

$$\{all\ buses\} = \{nodes\} + \{swing\}$$

$$V_n[0] \neq 0 \quad : \quad n \in \{all\}$$

Introduce power series expansion and compute order by order.

Order zero:

$$\sum_{n'\in\{nodes\}} Y_{nn'}V_{n'}[0] = -Y_{n,sw}V_{sw} + I_n \quad : \quad n\in\{nodes\}$$

requires non-singular admittance matrix $(Y)_{\{nodes\}\times\{nodes\}}$

All orders > 0

$N = 0, 1, \ldots$

$$\sum_{n'\in\{nodes\}} Y_{nn'}V_{n'}[N+1] = y_n^{(zip)}V_n[N] + S_n^* V_n^{(-1)*}[N] \quad : \quad n\in\{nodes\}$$

linear system with same matrix $(Y)_{\{nodes\}\times\{nodes\}}$

incremental : with $V_n[0 \ldots N+1]$ *compute* $V_n^{(-1)}[N+1]$ *and proceed to order* $N+2$

Construction of the Solution 2

- From the power series coefficients, compute the *N+1*-th truncation of the continued fraction (diagonal and near-diagonal Padé Approximants).

$$\frac{A_{N+1}^{(n)}(s)}{B_{N+1}^{(n)}(s)} \quad : \quad n \in \{nodes\}$$

- Check for error tolerance between the obtained values (at s=1) of successive continued fractions.

$$\left| \frac{A_{N+1}^{(n)}(1)}{B_{N+1}^{(n)}(1)} - \frac{A_{N}^{(n)}(1)}{B_{N}^{(n)}(1)} \right| \quad : \quad n \in \{nodes\}$$

A unique, deterministic solution is obtained as the analytic continuation of the power series from the s=0 operational state;
Or failure if no continuation from s=0 exists (unfeasible problem).

Enabling Capabilities

- This methodology enables:
  - To perform extensive Space-State searches for going from an operational state to another operational state. This allows solving difficult operation problems, such as the intelligent generation of action plans for correcting limit-violations, or restoration plans after a black-out.
  - When approaching voltage collapse, it allows spotting the weakest nodes, i.e. those that are responsible for the collapse (Sigma plots). It is possible to do so even if the case is already collapsed.

(just two examples)

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