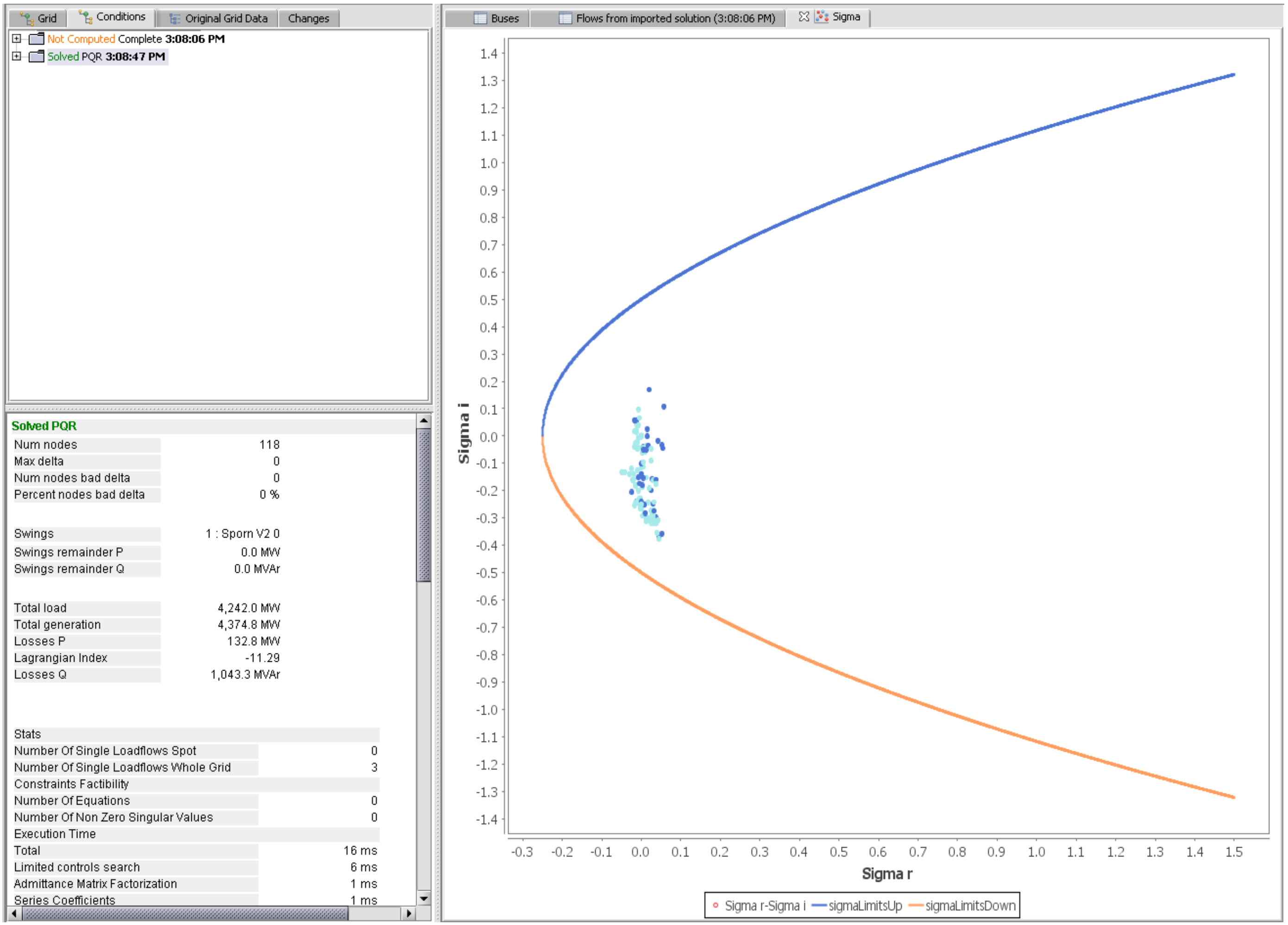
Sigma Plots

An Introduction

Sigma Plots

EleQuant



Default color convention:

- light blue dots represent PQ nodes (load buses).
- dark blue ones represent PV nodes.

The swing node is red and is always placed at the origin (0,0) of the coordinate system.

What are these Sigma values?

The Sigma Indicators and their corresponding Sigma Plots† are based on *Sigma Approximants*, a construct that under the HELM™ methodology performs a mathematical osculation of the load flow solution using a sort of (nonlinear) Thévenin equivalent.

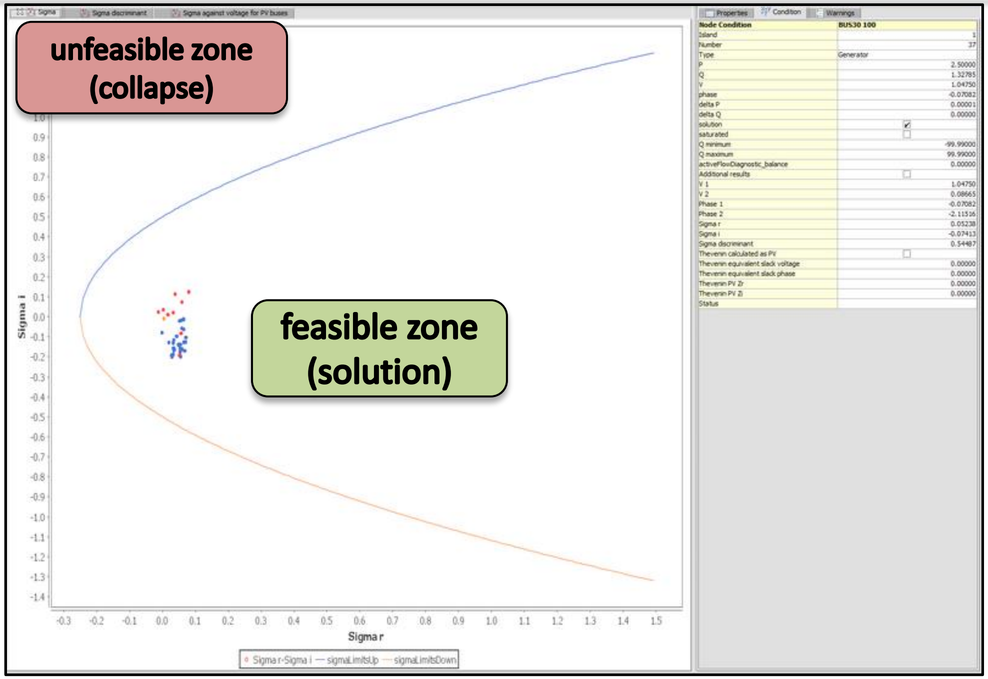
This equivalent consists in a two-bus model local to each bus, but where the swing V_{sw}=1 is *globally* the same:

$$V_i(s) = 1 + \frac{s\,S_i(S)}{V_i^*(s^*)}$$

- One complex value per node.
- They are dimensionless.
- They portray a global view of the condition of the whole network.
- And they have a key property: *they are well defined even beyond collapse!.*

† PATENT PENDING in the US.

What are they Good for?



They measure the distance of the system to voltage collapse, in a very graphical way:

- The case has a power flow solution if all sigma indicators are inside the parabola; and no solution (i.e. the case is collapsed) when there is one or more lying outside.
- Qualitatively, the closer the values get to the critical parabola, the closer the system is to collapse.
- When the case is collapsed, the buses whose sigma values lie outside the parabola are the ones most likely to be involved in the collapse.

Interpreting a Sigma Plot

We may interpret intuitively the position of points on a Sigma plot by thinking about the parameters of the two-bus equivalent:

- $\sigma_r = XQ + RP$
- $\sigma_i = XP - RQ$

where R+*j*X is an equivalent line impedance and P+*j*Q is the power injection at the bus.

In a transmission network (i.e. R/X <<1), we can approximate:

- $\sigma_r = XQ$
- $\sigma_i = XP$

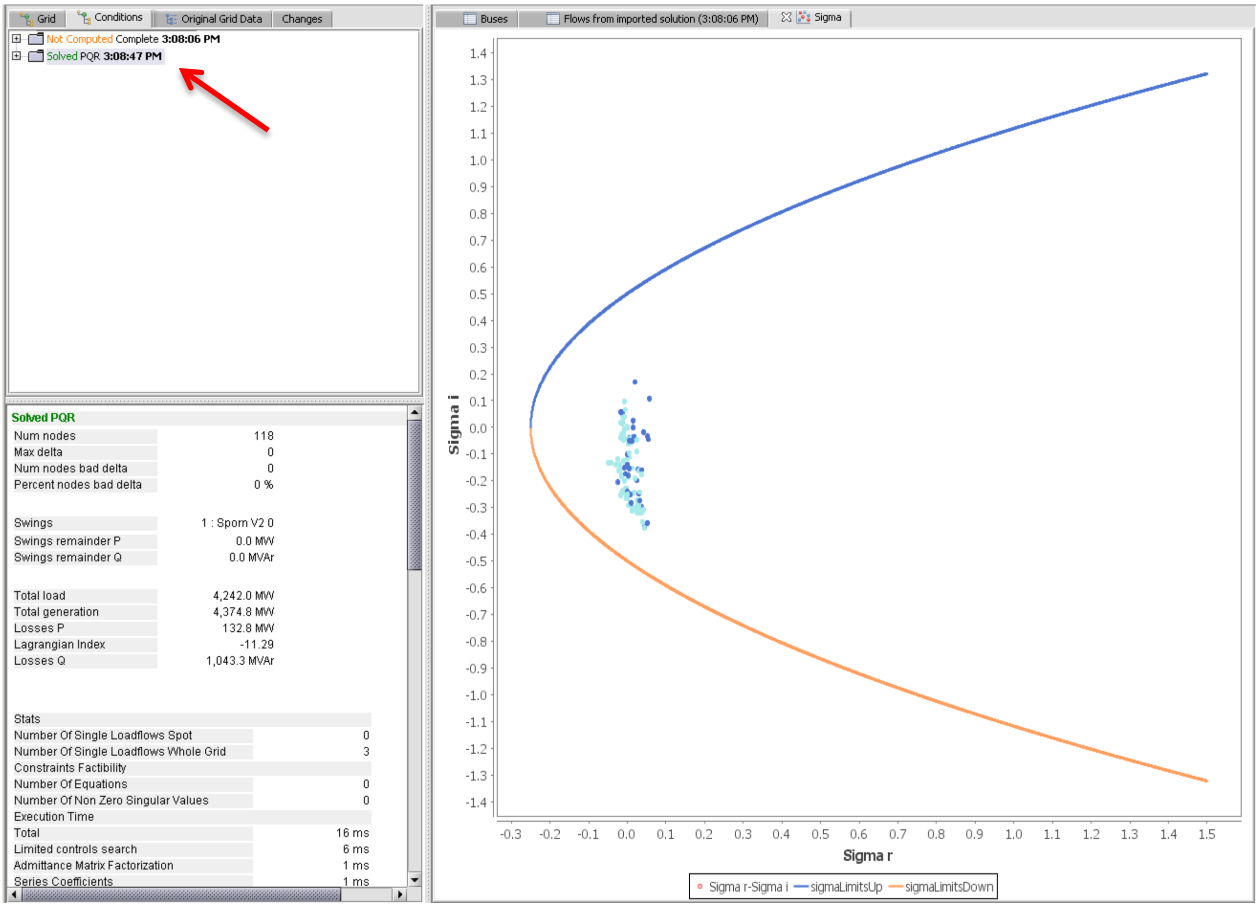
So on the vertical axis we may see two possible types of collapse (either too much active power generation or too much active power load), while on the horizontal axis the collapse is reached only by moving towards negative Q values (consumption of Q).

Interpreting a Sigma Plot

- Additionally, the Sigma Plot shows the nodes "sorted out" by their phase angle. This is a consequence of their definition (a global swing V=1 for all local equivalents).
- So the actual position of sigma values depends a lot on the **choice of swing**: the cloud of points will appear to rotate as the swing is moved around in the network.

However, the global behavior of the cloud of points, in what regards to their proximity to the critical parabola, remains basically the same.

Tracking Changes with Sigma Plots

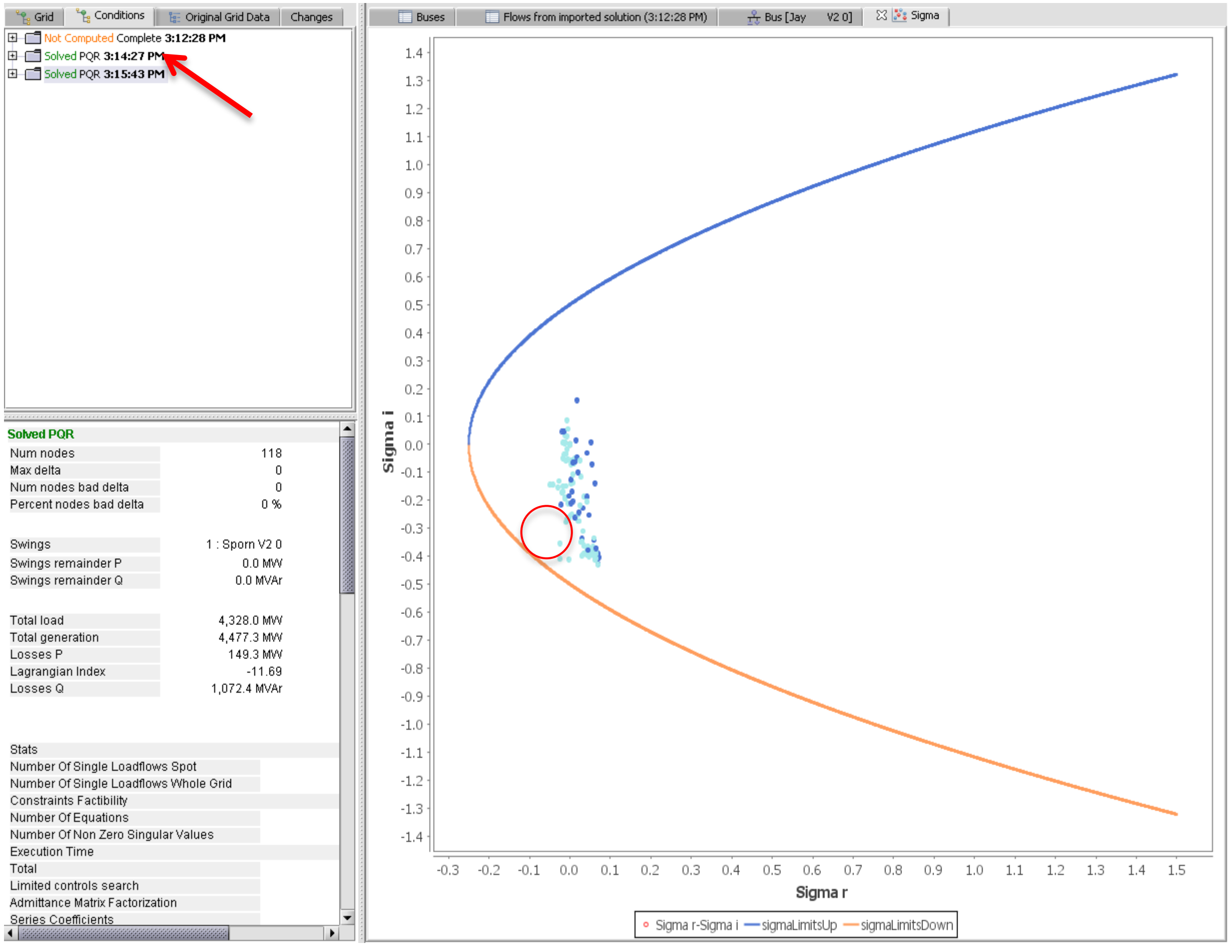


Model: IEEE118

Progressively increase the load at bus Jay V2 0.

We start at 50MW, 25MVAr.

Tracking Changes with Sigma Plots



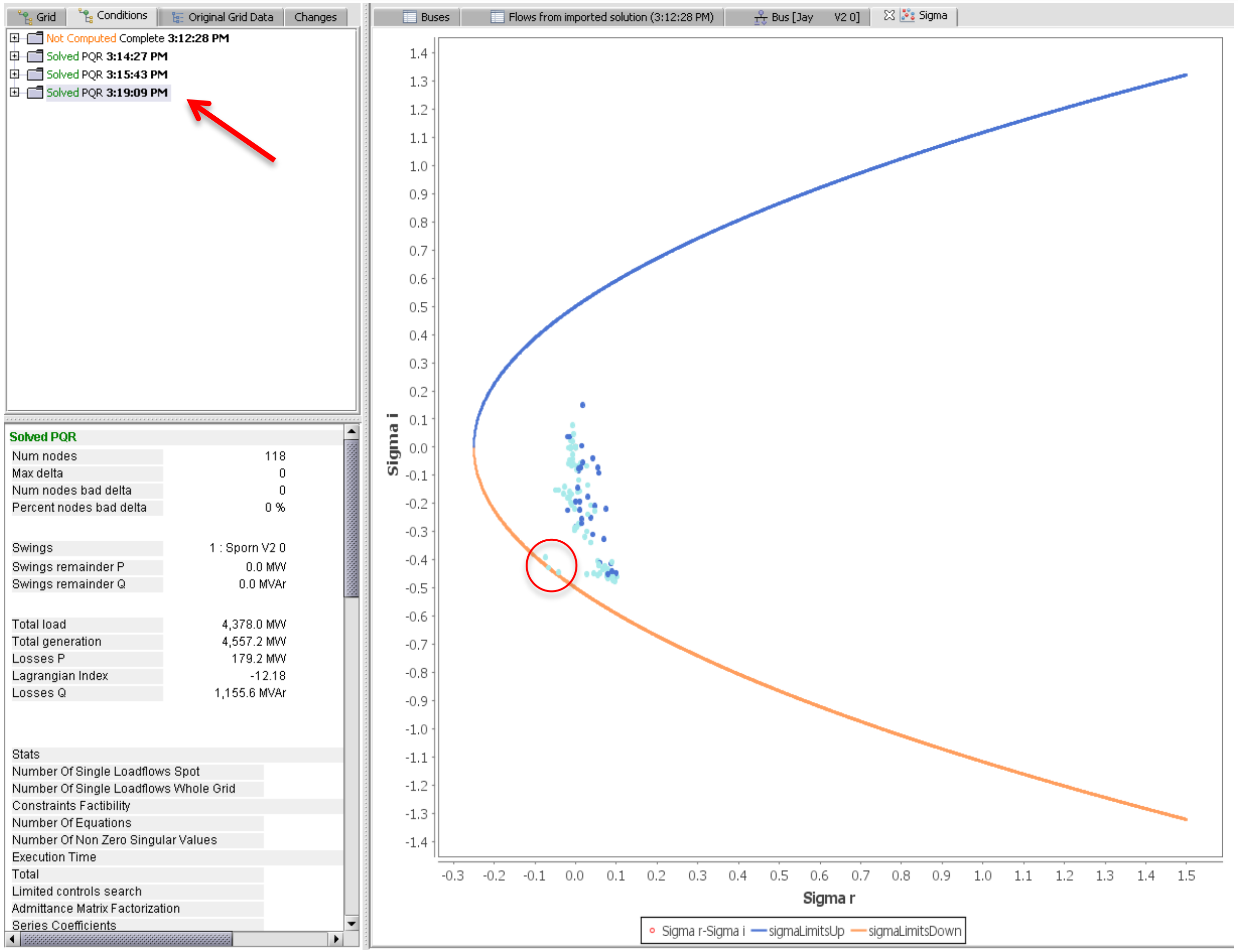
Model: IEEE118

Progressively increase the load at bus Jay V2 0.

Now 100MW, 50MVAr.

Note the three nodes straying away from the rest: they are Jay V2 0 and its two adjacent buses.

Tracking Changes with Sigma Plots



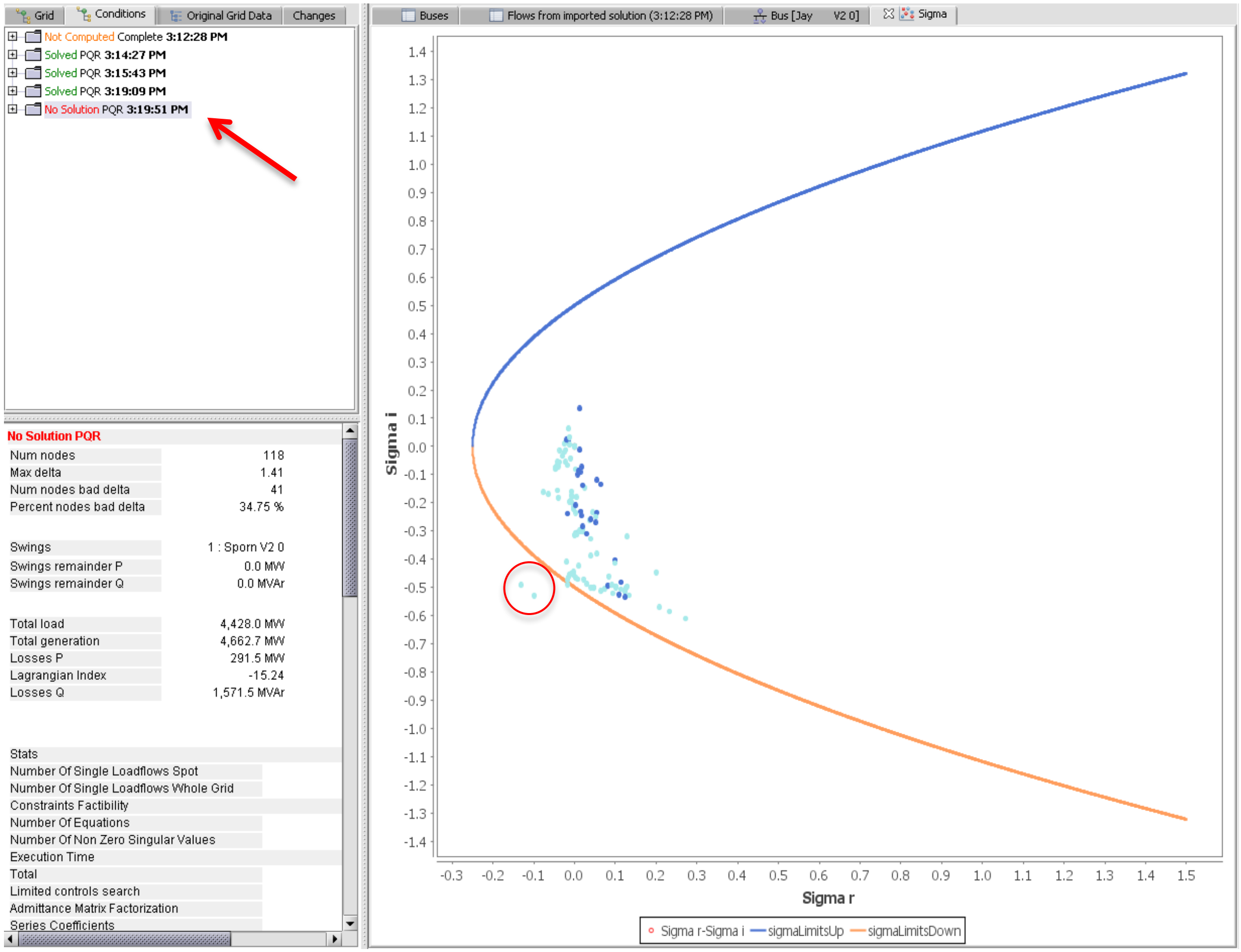
Model: IEEE118

Progressively increase the load at bus Jay V2 0.

Now 150MW, 100MVAr.

Now those nodes are very close to the feasibility boundary.

Tracking Changes with Sigma Plots



Model: IEEE118

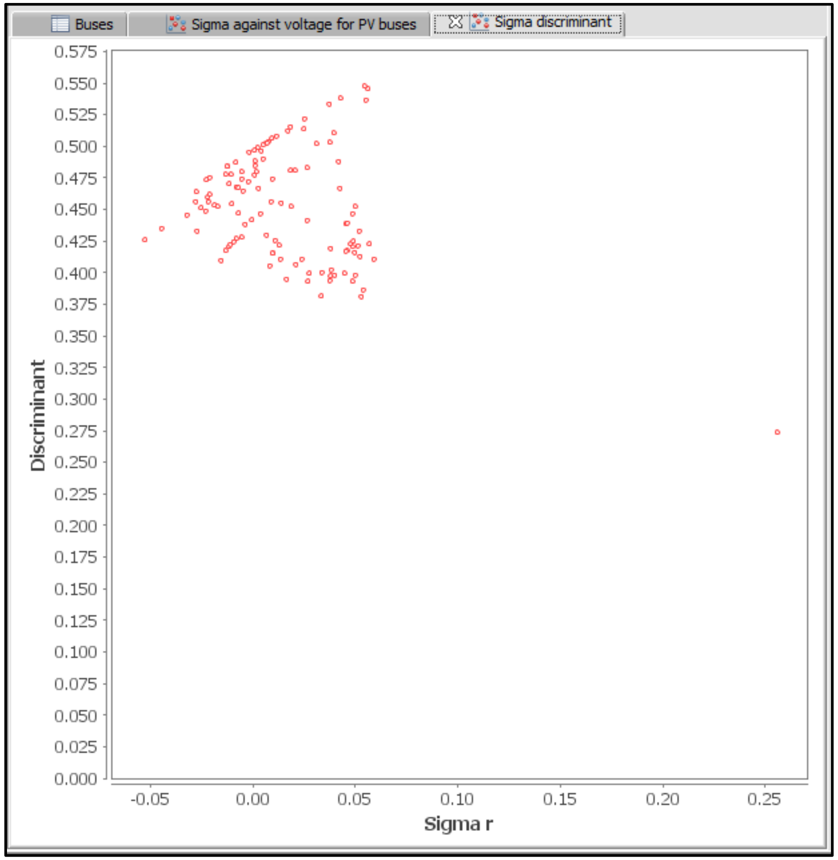
Progressively increase the load at bus Jay V2 0.

Now 200MW, 150MVAr.

COLLAPSE

Now some of those nodes have crossed the feasibility boundary.

Sigma Discriminant Chart



The Sigma Discriminant chart is another way to represent the same information shown in the standard Sigma Plot. It may be useful for **quantifying** the distance to collapse.

It plots the values of the discriminant $1/4-\sigma_i^2+\sigma_r$ versus σ_r (the real part of σ).

- positive values of the discriminant indicate that the node is within the feasibility region (inside the parabola)
- values approaching zero announce proximity to collapse.
- negative values indicate that the node is outside the feasibility region and therefore the power flow problem has no solution (i.e. it is unfeasible).

Sigma Plots (for PV nodes)

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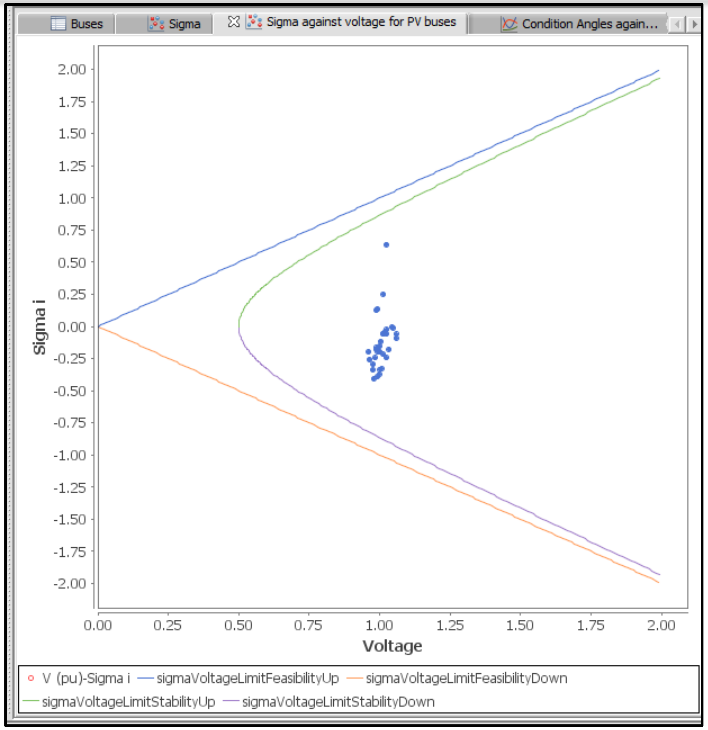

There is a dedicated Sigma Plot for PV nodes.

It plots the values of the imaginary part of σ versus the *voltage magnitude*.

- The boundary for collapse is now given by the two straight lines depicted in the chart: if all sigma points are inside the cone-shaped area, there is a power flow solution, whereas if there is one or more points outside then the system is collapsed (no solution).
- Additionally, a different parabola now marks the boundary for the *stability against the switch to PQ*. For the 2-bus local equivalent, PV nodes lying inside the parabola are stable, while those lying in the narrow region between the cone and the parabola are unstable. This instability has to be understood in the sense that, if changed into PQ type (while retaining the values of Q, voltage, and angle from the original solution), their voltage would belong to the non-operational branch. This signals that the V set-point for such nodes is too low (or, alternatively, P is too high).

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