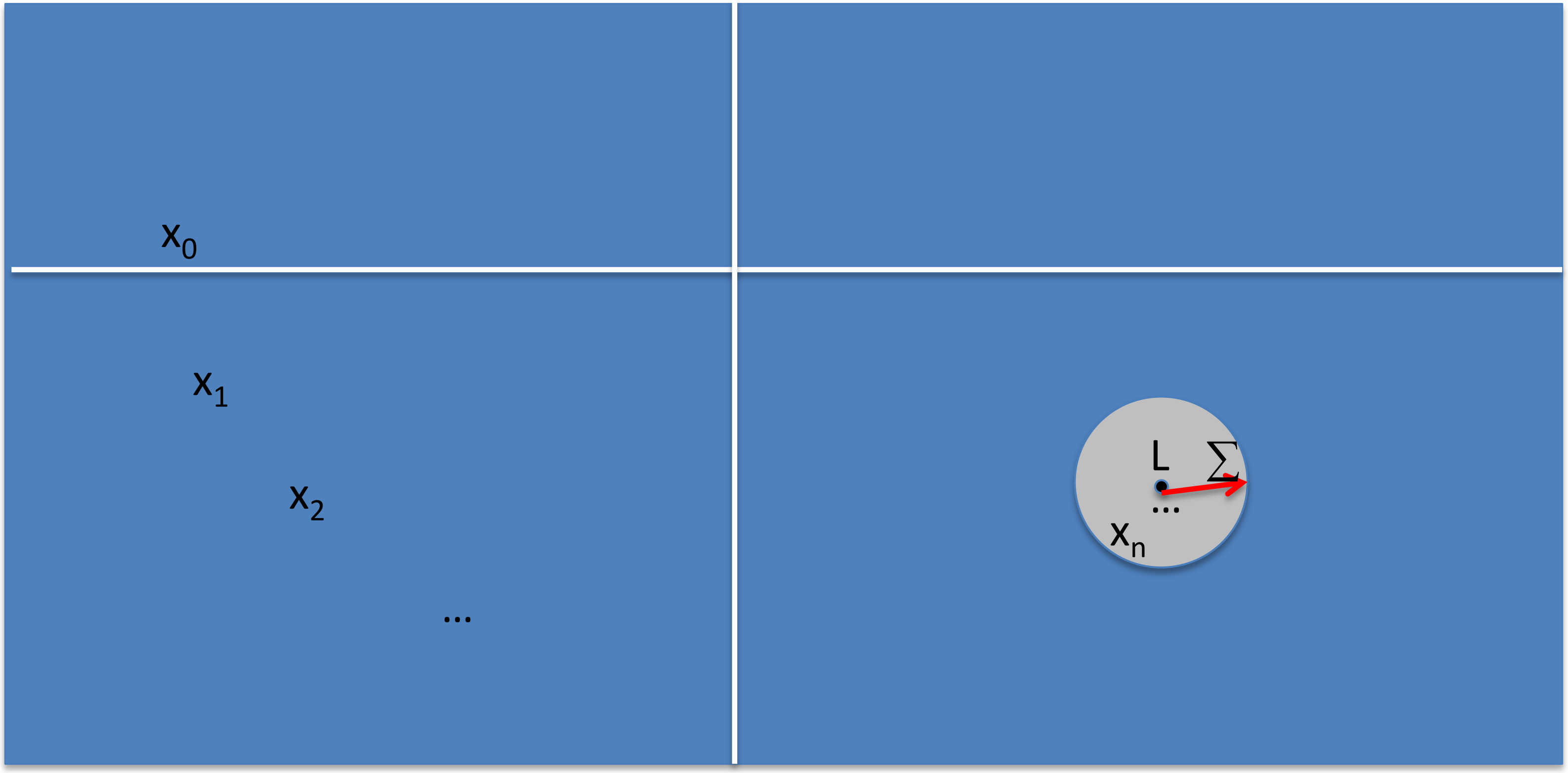
Solution and No Solution

Versus "Convergence and Non-Convergence"

Convergence

A sequence $x_0, x_1, .. x_n ....$ converges to L , or has limit L, if:



For any ε>0 we can find is an *n* such that $|L-x_m| < \varepsilon$ for all $m \geq n$

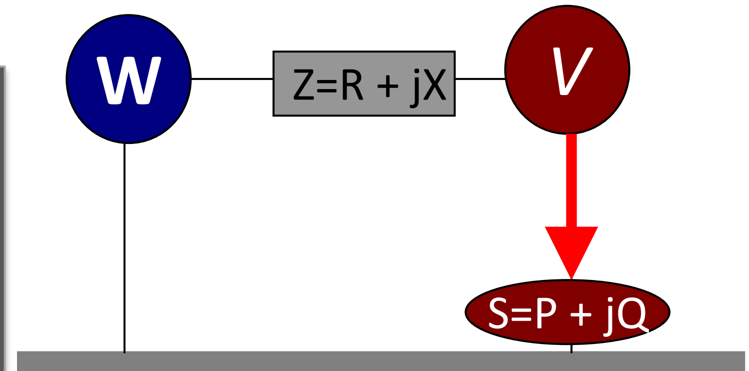
Convergence of Iterative Methods

- The basis of iterative methods is to start with a seed as a trial solution. The method provides a second trial solution. The procedure is iterated, and if the succession so produced has a limit then the method is said to converge.

- Remark: It is well known that a different seed may produce a non-convergent sequence, so the method in this case will not converge in spite of the existence of a solution to the problem.

- Example: Using naïve Newton-Raphson to find the roots of $x^3-4x^2+5x-2=0$, there is slow convergence to the root $x=1$, but the method fails to find the solution $x=2$.

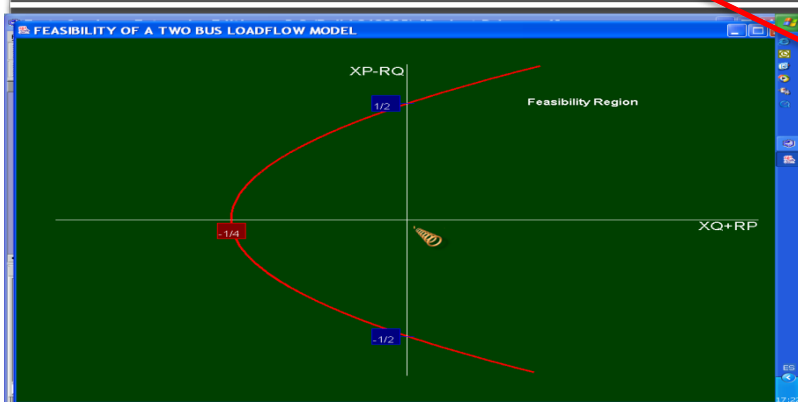
The Two Bus Case



$$V - W = ZI = Z\frac{S^*}{V^*} \qquad U \equiv \frac{V}{W}$$

$$U - 1 = \frac{ZS^*}{(WW^*)U^*} \qquad \sigma = \frac{ZS^*}{WW^*} \qquad U = 1 + \frac{\sigma}{U^*}$$

$$U = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \sigma_R - \sigma_I^2} + j\sigma_I$$



$$\sigma_R \geq -\frac{1}{4} + \sigma_I^2$$

- The two bus problem can be solved in closed form using elementary algebra. Its two solutions are displayed on the left.
- Note that for P=Q=0 (i.e. S=0 and therefore σ=0), the two solutions are U=1 and U=0.
- The operational solution is clearly the one with the (+) sign.

Iterative Solution of the Power Flow Problem: Newton Raphson

EleQuant

Newton Raphson and the two bus problem
http://www.elequant.com/technology/

Please note:

1) Iterative methods, even in the simplest case, may converge to a value that is not a solution of the problem, or be unable to converge when the solution exists.
2) The iterative methodology is unable to tell which of the two solutions is the operational one.

(black)	No convergence	The iterations have not convergend after the cut-off limit.
(dark gray)	Anomalous convergence	The iterations have converged to a point that is neither one of the two known solutions.
(red)	Unstable: wrong winding	The iterations have converged to the unstable solution, but the angle is shifted by possibly large multiples of 2π.
(dark red)	Unstable: right winding	The iterations have converged to the unstable solution, and the angle is normalized within -π, π.
(gray)	Stable: wrong winding	The iterations have converged to the correct operational solution, but the angle is shifted by possibly large multiples of 2π.
(light gray)	Stable: right winding	The iterations have converged to the correct operational solution, and the angle is normalized within -π, π.

HELM Finds the Solution

EleQuant

It can be explicitly shown by direct computation in the two-bus case that HELM builds exactly the algebraic solution:

$$U = \frac{1}{2} + \sqrt{\frac{1}{4} + \sigma_R - \sigma_I^2 + j\sigma_I}$$

to the desired order of accuracy, through Padé Approximants.

If the values of σ belong to the feasibility region, the method always finds the operational solution; if the values are outside this region, it finds none as there is no solution (the Padé Approximants oscillate).

a) Notice the (+) sign in the square root, which corresponds to the operational branch.
b) The same is true in the general N-node case, although the solution cannot be shown in closed form like in this simple example.

HQ Barcelona
Av. de la Torre Blanca, 57
08172 Sant Cugat del Vallès
Barcelona
Tel. +34 93 504 49 00

San Francisco
48 Terra Vista Ave. # D
San Francisco, CA 94115
Tel. 1 415 978 98 00
Fax. 1 415 978 98 10