

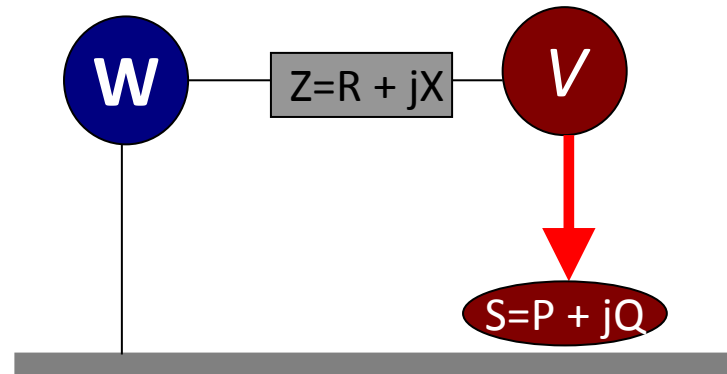


The Operational Solution versus Non-Operational Solutions

Which Solutions are Really
Possible in a Real Network?



- By 'Operational Solution' we mean the kind of solution that all the control devices in the network aim to stabilize.
- Most voltage control devices in power networks work by injecting reactive power when detecting that the voltage decreases. This is indeed the physical behavior under a normal operating conditions, i.e. the voltage magnitude grows when injecting more reactive power.
- If we had a solution where the voltage decreased when injecting reactive power, the voltage control devices would be unstable: under the slightest voltage drop, they would inject more reactive power, causing the voltage to decrease further, thus provoking a voltage collapse.
- These kind of solutions are termed non-operational because they can not survive in actual electrical networks, as they are designed to operate the other way around.



- Ohm's law $V - W = ZI = Z \frac{S^*}{V^*}$
- Define $U \triangleq \frac{V}{W}; S \triangleq S \frac{Z}{W^* W^*}$
- Rewrite Ohm's law as: $U = 1 + \frac{S}{U^*}$



- Multiply by U^* : $U^*U^* = |U|^2 = U^* + S$
- Separate imaginary and real parts: $|U|^2 = U_R + S_R; U_I = S_I$
- Second order eqn. for U_R : $U_R^2 - U_R + S_I^2 - S_R = 0$
- Where $S_R = \frac{RP + XQ}{|W|^2}; S_I = \frac{XP - RQ}{|W|^2}$
- Solutions: $U_R = \frac{1}{2} \pm \sqrt{\frac{1}{4} + \sigma_R - \sigma_I^2}$ provided $\frac{1}{4} + S_R - S_I^2 \geq 0$

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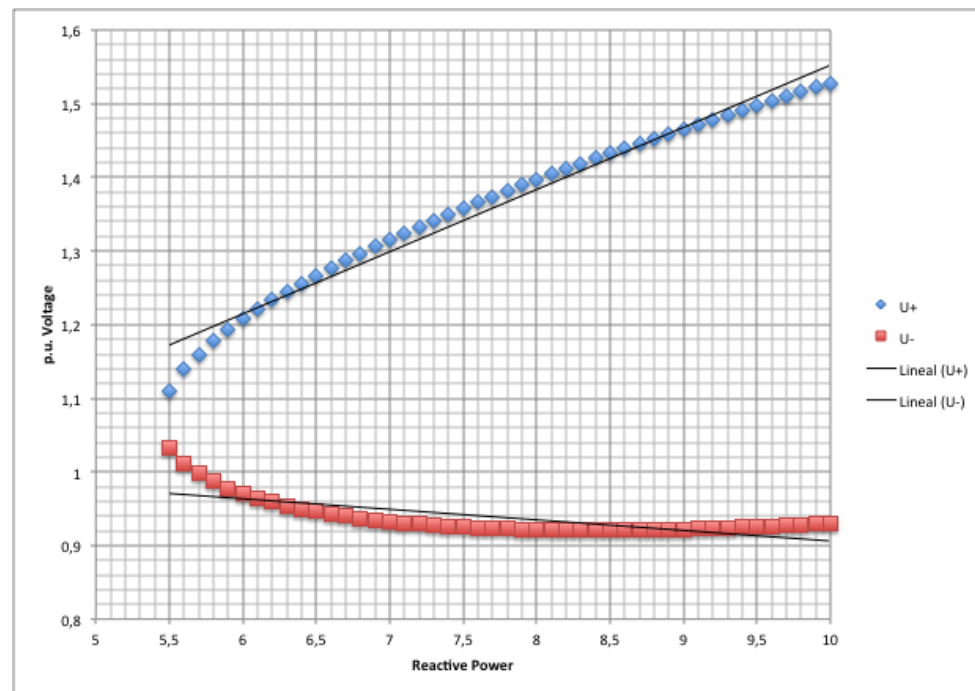
Consider the case where $R=X=0$, in this case $\sigma=0$ and the two solutions reduce to:

- $U_+ = 1$
- $U_- = 0$

Where the sub-indexes indicate the choice of the sign in the square root. The behavior one seeks for a solution is clearly the one displayed by U_+ .

The graph on the right displays the behavior of these two solutions as Q varies for the following values of the parameters:
 $R=0.01$, $X=0.1$, $P=10\text{MW}$

In the solution U_+ it becomes apparent that the voltage increases with Q , whereas the opposite happens to U_- .





- The power series used by HELM are of the form:

$$V_i(s) = V_{sw} + sV_i^1[0] + \dots + s^n V_i^n[0] + \dots; i = [1, N]$$

- Where V_{sw} stands for the swing voltage; this guarantees that the solution is linked to U_+ at each node.
- For PV nodes a conceptually similar embedding condition is used, to fulfill the constraint on $|V|$ at these nodes.
- This singles out from all the possible solutions the unique solution that is compatible with the voltage controls existing on the electric network.
- Thus the solution HELM provides is precisely the operational solution.



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